PHY optimisation with uncertainty

Applications in spectrum sharing scenarios

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The considered spectrum sharing and uncertainty scenarios

- Cognitive radio and Licensed/Authorized Shared Access
- Non-Orthogonal Multiple Access Schemes

- How sure can we be that the spectrum is not used by its intended user? (CR)
- Given the concurrent operation of multiple systems, how much information can I assume that I have concerning the interference that I cause to other transmissions? (CR, NOMA)
- How much information can I have concerning the interference that other users cause to me? (CR, NOMA)
- How can we model all the above sources of uncertainty and include them in our PHY and MAC decision making process? (CR, NOMA)

Cognitive radio: Information structures for modelling uncertainty and how to use them

Cognitive radio systems

- Interweave cognitive radio: A secondary/cognitive user (SU/CU) transmits by exploiting spatio-temporal "spectrum wholes", i.e., time instances/locations where some frequency band is not used by its intended user. Spectrum sensing is required in order to determine whether or not a primary user is present.
- Underlay cognitive radio: A SU/CU transmits in the presence of primary communication, provided that the interference caused to primary reception is below a pre-determined interference temperature.

Modifying the traditional CR design:

 Hybrid cognitive radio: After spectrum sensing is applied, depending on whether a primary user has been detected or not, we apply interweave and or underlay.

Today's talk: How to explore Channel State Information (CSI) availability structures which encompass uncertainty (i.e. mixed CSI availability scenarios) in order to :

- Introduce CR designs based on more meaningful QoS constraints instead fo a simple interference temperature constraint.
- Optimize the several decisions (e.g., power allocation, beamforming, sensing time) for the considered mixed CSI scenarios, in the presence of further uncertainties introduced by imperfect sensing.

The considered mutli-antenna uplink interference model



Figure 1: The considered system model

Rayleigh fading assumption:

- PTx PRx channel $\mathbf{h}_{1,1} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_{1,1})$
- PTx SRx channel $\mathbf{h}_{1,2} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{R}_{1,2}\right)$
- STx-PRx channel $\mathbf{h}_{2,1} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{R}_{2,1}\right)$
- STx SRx channel $\textbf{h}_{2,2} \sim \mathcal{CN}\left(\textbf{0},\textbf{R}_{2,2}\right)$

The considered uplink information availability scenario



Figure 2: The considered CSI availability structure

The exact mode of operation

Operation of the Primary System

- User₁ transmits with probability \mathcal{P}_1 .
- User1 transmits using a power level P1 and it is agnostic of whether secondary user transmits or not.
- BS₁ uses a Maximum Ratio Combining (MRC) receiver in order to maximize the received power that corresponds to the transmission of User₁.

Operation of the Secondary System

- User₂ first senses the wireless channel in order to determine whether a user is present or not.
- If no user is detected, User₂ transmits using full power P_{max}. BS₂ employs an MRC receiver such as to maximize the received power that corresponds to the transmission of User₂.
- Sensing is designed such as to make sure that primary user presence is detected with very high probability.
- If a user is detected, $User_2$ transmits using a power level $P_{\mathit{und}} \leq P_{\mathit{max}}.$
- P_{und} is selected such as to make sure that an outage probability constraint is satisfied for User₁.
- BS₂ applies a beamformer w₂ designed such as to maximize some measure of achievable rate which can be calculated using the available Channel State Information.
- Sensing time is also optimized such as to maximize the same measure of achievable rate.

The transmission frame structure



Figure 3: Structure of the secondary user transmission frame

- The more we sense, the more certain we can be about our decision about spectrum use. However, we reduced the opportunities to transmit.
- We assume that spectrum sensing is applied at *User*₂, and the fading channel between *User*₁ and *User*₂ is a Single Input Single Output (SISO) Rayleigh fading channel.

The spectrum sensing problem: energy detector

- If the energy of the signal during the first \(\tau\) seconds of the frame is above a threshold, we decide upon the presence of a user, if not, we decide that no primary user is present.
- Signal model: The sampled version of the signal obtained during the sensing period is written as:

$$y_{s}[n] = \begin{cases} \omega[n], & \text{If no primary transmission is present} \\ \sqrt{P_{1}}h_{0}s_{1}[n] + \omega[n], & \text{otherwise} \end{cases}$$
(1)

where:

- ω[n] ~ CN (0, N₀) the AWGN at User2.
- $h_0 \sim \mathcal{CN}(0, \tilde{g}_0)$ the Single Input Single Output (SISO) sensing Rayleigh fading channel.
- P₁ the transmit power of the primary user (if present).
- Decision metric and decision process:

$$y = \sum_{n=1}^{N} |y_{s}[n]|^{2} \gtrless \epsilon = \begin{cases} A \text{ user is present} \\ No \text{ user is present} \end{cases}$$
(2)

- Missed detection: The event that we decide that no user is present, given that in reality a user is present.
- False alarm: The event that we decide that a user is present, given that in reality no user is present.

Approximating the performance analysis of the sensing process

• The decision variable:

$$y = \sum_{n=1}^{N} |y_{s}[n]|^{2} \ge \epsilon = \begin{cases} \text{A user is present} \\ \text{No user is present} \end{cases}$$
(3)

- Applying the central limit theorem:
 - False alarm probability:

$$P_{fa} = \Pr(y > \epsilon | \text{no user is present}) = Q\left(\sqrt{N}\left(\frac{\epsilon}{N_0} - 1\right)\right)$$
(4)

• Conditional detection probability:

$$P_{d|h_0} = \Pr\left(y > \epsilon \mid h_0, \text{ A user is present}\right) = Q\left(\sqrt{N}\left(\frac{\epsilon}{|h_0|^2 P_1 + N_0} - 1\right)\right)$$
(5)

• Detection probability¹:

$$P_{d} = \mathbb{E}_{h_{0}}\left\{P_{d|h_{0}}\right\} \approx Q\left(\sqrt{N}\left(\frac{\epsilon}{\mathbb{E}\left\{|h_{0}|^{2}\right\}P_{1}+N_{0}}-1\right)\right)$$
(6)

• Q: The Gaussian Q function.

¹M. C. Filippou, D. Gesbert and G. A. Ropokis, "A comparative performance analysis of interweave and underlay multi-antenna cognitive radio networks", IEEE Transactions on Wireless Communications, vol. 14, no. 5, pp. 2912-2925, May 2015

Mathematically formulating our system design

- Objective: Appropriately select:
 - the power level P_{und}
 - the beamformer \boldsymbol{w}_2
 - the sensing time τ
 - The energy detection threshold ϵ

such as to maximize the rate of secondary user User₂.

- Constraint 1: Select the detection threshold ϵ such as to ensure that a target detection probability is met.
- Constraint 2: Protect the primary user by making sure that the primary user outage probability is below a threshold.
- Necessary steps:
 - 1. Using the (missed) detection probability such as to find a closed form expression for the outage probabilty.
 - Find closed form expressions for the average (with respect to interference) rate of secondary communication.
 - Use the derived expressions to solve the resulting rate optimization problem subject to the outage probability constraint and the target detection probability constraint.

Step 1: Evaluating the outage probability for primary transmission

• Assumption: Primary user is employing a maximum ratio combiner:

$$\mathbf{w}_1 = \frac{\mathbf{h}_1}{\|\mathbf{h}_1\|},\tag{7}$$

• The primary user outage probability is expressed as:

$$P_{out} = \underbrace{P_d P_{out}^{(1)}}_{eut} + \underbrace{(1 - P_d) P_{out}^{(0)}}_{out}$$
(8)

Primary user detected Primary user missed

• The signal reaching BS₁ in case User₁ is present, is expressed as:

$$y_{1}(t) = \mathbf{w}_{1}^{H} \mathbf{h}_{1,1} \sqrt{P_{1}} s_{1}(t) + \mathbf{w}_{1}^{H} \mathbf{h}_{2,1} \sqrt{P_{und}} s_{2}(t) + \eta_{t}$$

$$= \|\mathbf{h}_{1,1}\| \sqrt{P_{1}} s_{1}(t) + \frac{\mathbf{h}_{1,1}^{H} \mathbf{h}_{2,1}}{\|\mathbf{h}_{1,1}\|} \sqrt{P_{und}} s_{2}(t) + \eta(t).$$
(9)

• Primary user SINR (if its transmission is detected):

$$\gamma_{1} = \frac{\|h_{1,1}\|^{2} P_{1}}{N_{0} + P_{und} \frac{|\mathbf{h}_{1,1}^{H} \mathbf{h}_{2,1}|^{2}}{\|\mathbf{h}_{1,1}\|^{2}}}$$
(10)

• Given $\mathbf{h}_{1,1}$: $P_{out|\mathbf{h}_{1,1}}^{(1)}(P_{und}) = \Pr(\gamma_1 \le \gamma_0 | \mathbf{h}_{1,1}, P_{und})$

Step 1: Evaluating the outage probability of primary transmission

• Introducing $Y = \frac{\left|\mathbf{h}_{1,1}^{H}\mathbf{h}_{2,1}\right|^{2}}{\left|\left|\mathbf{h}_{1,1}\right|\right|^{2}}$, we equivalently have:

$$P_{out|\mathbf{h}_{1,1}}^{(1)}(P_{und}) = \Pr\left(Y \ge \frac{\|\mathbf{h}_{1,1}\|^2 P_1}{\gamma_0 P_{und}} - \frac{N_0}{P_{und}}|\mathbf{h}_{1,1}\right).$$
 (11)

- Due to correlated Ralyeigh fading: $h_{2,1}=R_{2,1}^{1/2}h_{2,1,w}$ where $h_{2,1,w}\sim\mathcal{CN}\left(0,I\right)$.
- Given $h_{1,1}$, variable $Y = \frac{\left|h_{1,1}^{H}R_{2,1}^{1/2}h_{2,1,w}\right|^{2}}{\left\|h_{1,1}\right\|^{2}}$ is expressed as the square of a linear combination of independent complex Gaussian Random variables.
- Variable Y is an exponentially distributed random variable with conditional expectation:

$$\mathbb{E}\left\{Y|\mathbf{h}_{1,1}\right\} = \frac{\left|\mathbf{h}_{1,1}^{H}\mathbf{R}_{2,1}^{1/2}\right|^{2}}{\left\|\mathbf{h}_{1,1}\right\|^{2}} = \frac{\mathbf{h}_{1,1}^{H}\mathbf{R}_{2,1}\mathbf{h}_{1,1}}{\left\|\mathbf{h}_{1,1}\right\|^{2}}.$$
 (12)

• We can express the conditional outage probability as:

$$P_{out|\|\mathbf{h}_{1,1}\|^{2}}^{(1)}(P_{und}) = \exp\left(-\frac{\left\|\mathbf{h}_{1,1}^{2}P_{1}\right\|^{2}}{\gamma_{t}P_{und}\mathbb{E}\left\{Y|\mathbf{h}_{1,1}\right\}} + \frac{N_{0}}{P_{und}\mathbb{E}\left\{Y|\mathbf{h}_{1,1}\right\}}\right).$$
 (13)

• In order to derive a closed form approximation for this expression, we substitute $\mathbb{E} \{ Y | \mathbf{h}_{1,1} \}$ by its expectation (with respect to $\mathbf{h}_{1,1}$), which is found as an expectation of a ratio of quadratic forms in complex circularly symmetric normal random variables, and can be written in a convenient closed form.

•
$$P_{out}^{(1)}(P_{und}) = \mathbb{E}_{\|\mathbf{h}_{1,1}\|^2} \left\{ \tilde{P}_{out\|\mathbf{h}_{1,1}\|^2}(P_{und}) \right\}$$

• Similarly, we can approximate $P_{out}^{(0)}$ as:

$$P_{out}^{(0)}(P_{max}) = \mathbb{E}_{\|\mathbf{h}_{1,1}\|^2} \left\{ \tilde{P}_{out}^{(1)}\|_{\mathbf{h}_{1,1}\|^2}(P_{max}) \right\}.$$
(14)

Step 1: Evaluating the outage probability for primary transmission

Assuming a system with ${\it M}$ transmit antennas at each BS and an exponential correlaction model.



Figure 4: Evaluating the quality of the outage probability approximation²

²M. C. Filippou, G. A. Ropokis, D. Gesbert and T. Ratnarajah, "Joint Sensing and Reception Design of SIMO Hybrid Cognitive Radio Systems", IEEE Transactions on Wireless Communications, vol. 15, no. 9, September 2016

Evaluating the rate of secondary communucation

• In the absence of a primary transmission, assuming no sensing error:

$$R^{(0,0)} = \log_2\left(1 + \frac{\|\mathbf{h}_{2,2}\|^2 P_{max}}{N_0}\right)$$
(15)

• In the absence of a primary transmission and with a false alarm event occuring:

$$R^{(0,1)} = \log_2 \left(1 + \frac{|\mathbf{w}_2^H \mathbf{h}_{2,2}|^2 P_{und}}{N_0} \right)$$

$$= \log_2 \left(\frac{\mathbf{w}_2^H \mathbf{H}_{eff} \mathbf{w}_2}{\|\mathbf{w}_2\|^2} \right), \text{ with } \mathbf{H}_{eff} = \mathbf{I} + \frac{P_{und}}{N_0} \mathbf{h}_{2,2} \mathbf{h}_{2,2}^H.$$
(16)

• In the presence of a primary transmission, with a missed detection:

$$R^{(1,0)} = \log_2 \left(1 + \frac{\|\mathbf{h}_{2,2}\|^2 P_{max}}{\frac{\|\mathbf{h}_2^H \mathbf{h}_{1,2}\|^2 P_1}{\|\mathbf{h}_{2,2}\|^2} + N_0} \right)$$
(17)

• In the presense of primary transmission, with correct detection:

$$R^{(1,1)} = \log_2\left(1 + \frac{|\mathbf{w}_2^H \mathbf{h}_{2,2}|^2 P_{und}}{|\mathbf{w}_2^H \mathbf{h}_{1,2}|^2 P_1 + N_0}\right)$$
(18)

The final rate expression

$$R = \frac{T - \tau}{T} \times \left(\mathcal{P}_0 \left(1 - P_{fa} \left(\tau, \epsilon \right) \right) R^{(0,0)} + \mathcal{P}_0 P_{fa} \left(\tau, \epsilon \right) \right) R^{(0,1)} \left(\mathbf{w}_2, P_{und} \right) + \mathcal{P}_1 \left(1 - P_d \left(\tau, \epsilon \right) \right) R^{(1,0)} + \mathcal{P}_1 P_d \left(\tau, \epsilon \right) R^{(1,1)} \left(\mathbf{w}_2, P_{und} \right) \right)$$
(19)

The final rate expression

$$R = \frac{T - \tau}{T} \times \left(\mathcal{P}_{0} \left(1 - P_{f_{a}} \left(\tau, \epsilon \right) \right) R^{(0,0)} + \mathcal{P}_{0} P_{f_{a}} \left(\tau, \epsilon \right) \right) R^{(0,1)} \left(P_{und}, \mathbf{w}_{2} \right) + \mathcal{P}_{1} \left(1 - P_{d} \left(\tau, \epsilon \right) \right) R^{(1,0)} + \mathcal{P}_{1} P_{d} \left(\tau, \epsilon \right) R^{(1,1)} \left(\mathbf{w}_{2}, P_{und} \right) \right)$$
(19)

Step 2: Averaging over uncertainty

• In the absence of instantaneous information concerning h_{1,2}, we can use as a performance measure the expectation:

$$\mathbb{E}_{\mathbf{h}_{1,2}}\left\{R^{(1,0)}\right\} = \mathbb{E}_{\mathbf{h}_{1,2}}\left\{\log_{2}\left(1 + \frac{\|\mathbf{h}_{2,2}\|^{2} P_{max}}{\frac{\left|\mathbf{h}_{2,2}^{H}\mathbf{h}_{1,2}\right|^{2} P_{1}}{\|\mathbf{h}_{2,2}\|^{2}} + N_{0}}\right)\right\}$$
(20)

- Exploiting the assumption of correlated Rayleigh fading, we have that: $h_{1,2} = R_{1,2}^{1/2} h_{1,2,w} \text{ where } h_{1,2,w} \sim \mathcal{CN}(0,I).$
- Given $\mathbf{h}_{2,2}$, variable $\mathbf{h}_{2,2}^{H}\mathbf{h}_{1,2}$ is a complex Gaussian random variable, with variance $\left|\mathbf{h}_{2,2}^{H}\mathbf{R}^{1/2}\right|^{2} = \mathbf{h}_{2,2}^{H}\mathbf{R}_{1,2}\mathbf{h}_{2,2}$.
- Using Jensen's bound, we can bound $\mathbb{E}_{h_{1,2}}\left\{ {{R}^{\left(1,0 \right)}} \right\}$ as:

$$\mathbb{E}_{\mathbf{h}_{1,2}}\left\{R^{(1,0)}\right\} \ge \log_{2}\left(1 + \frac{\|\mathbf{h}_{2,2}\|^{2} P_{max}}{\frac{\mathbf{h}_{2,2}^{H} \mathbf{R}_{1,2} \mathbf{h}_{2,2} P_{1}}{\|\mathbf{h}_{2,2}\|^{2}} + N_{0}}\right) = \tilde{R}^{(1,0)}.$$
 (21)

Step 2: Averaging over uncertainty

• In the absence of instantaneous informaction concerning $\mathbf{h}_{1,2}$ we can use as a performance measure the expectation:

$$\mathbb{E}_{\mathbf{h}_{1,2}} \left\{ R^{(1,1)} \right\} = \mathbb{E}_{\mathbf{h}_{1,2}} \left\{ \log_2 \left(1 + \frac{|\mathbf{w}_2^H \mathbf{h}_{2,2}|^2 P_{und}}{|\mathbf{w}_2^H \mathbf{h}_{1,2}|^2 P_1 + N_0} \right) \right\}$$

$$= \mathbb{E}_{\mathbf{h}_{1,2}} \left\{ \log_2 \left(\frac{\mathbf{w}_2^H \mathbf{H}_{eff} \mathbf{w}_2}{||\mathbf{w}_2||^2} \right) \right\}$$

$$- \mathbb{E} \left\{ \log_2 \left(1 + \frac{P_1}{N_0} \mathbf{w}_2^H \mathbf{h}_{1,2} \mathbf{h}_{1,2}^H \mathbf{w}_2 \right) \right\}$$

$$(22)$$

• Using the gaussianity of $h_{1,2}$ and the above correlation model, we obtain:

$$\begin{split} \mathcal{E}_{h_{1,2}}\left\{R^{(1,1)}\right\} &= \log_{2}\left(\frac{\mathbf{w}_{2}^{H}\mathbf{H}_{eff}\,\mathbf{w}_{2}}{\|\mathbf{w}_{2}\|^{2}}\right) \\ &+ \frac{1}{\ln 2}\exp\left(\frac{\mathbf{w}_{2}^{H}\mathbf{H}_{eff}\,\mathbf{w}_{2}}{\mathbf{w}_{2}^{H}\frac{P_{1}\mathbf{R}_{1,2}}{N_{0}}\mathbf{w}_{2}}\right)E_{1}\left(\frac{\mathbf{w}_{2}^{H}\mathbf{H}_{eff}\,\mathbf{w}_{2}}{\mathbf{w}_{2}^{H}\frac{P_{1}\mathbf{R}_{1,2}}{N_{0}}\mathbf{w}_{2}}\right) \\ &- \frac{1}{\ln 2}\exp\left(\frac{1}{\frac{P_{1}}{N_{0}}\mathbf{w}_{2}^{H}\mathbf{R}_{1,2}\mathbf{w}_{2}}\right)E_{1}\left(\frac{1}{\frac{P_{1}}{N_{0}}\mathbf{w}_{2}^{H}\mathbf{R}_{1,2}\mathbf{w}_{2}}\right) \end{split}$$
(23)

• Recall:

$$\mathbf{H}_{eff} = \mathbf{I} + \frac{P_{und}}{N_0} \mathbf{h}_{2,2} \mathbf{h}_{2,2}^H$$
(24)

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The optimal system design problem

$$\begin{aligned} \underset{P_{und},\tau,\mathbf{w}_{2},\epsilon}{\text{maximize:}} & R = \frac{T-\tau}{T} \times \left(\mathcal{P}_{0} \left(1 - P_{fa} \left(\tau, \epsilon \right) \right) R^{(0,0)} \right. \\ & \left. + \mathcal{P}_{0} P_{fa} \left(\tau, \epsilon \right) \right) R^{(0,1)} \left(\mathbf{w}_{2}, P_{und} \right) \\ & \left. + \mathcal{P}_{1} \left(1 - P_{d} \left(\tau, \epsilon \right) \right) \tilde{R}^{(1,0)} \right. \\ & \left. + \mathcal{P}_{1} P_{d} \left(\tau, \epsilon \right) \mathbb{E}_{h_{1,2}} \left\{ R^{(1,1)} \left(\mathbf{w}_{2}, P_{und} \right) \right\} \right) \end{aligned}$$
(25)
subject to: $P_{d} = P_{d}^{(target)} \\ P_{out} = P_{out}^{(target)}, \\ & 0 \leq P_{und} \leq P_{max}, \\ & \left\| \mathbf{w}_{2} \right\| = 1. \end{aligned}$

• For fixed sensing parameters, selecting P_{und} reduces to solving the equation:

$$P_{out} = P_{out}^{(target)}.$$
 (26)

• For fixed *P_{und}* and beamformer, solving the optimal sensing problem reduces to solving the problem:

$$\begin{array}{l} \underset{\tau,\epsilon}{\text{maximize:}} \\ R = \frac{T - \tau}{T} \times \\ \left(\mathcal{P}_0 \left(1 - P_{fa} \right) R^{(0,0)} + \mathcal{P}_0 P_{fa} R^{(0,1)} + \mathcal{P}_1 \left(1 - P_d \right) \tilde{R}^{(1,0)} + \mathcal{P}_1 P_d R^{(1,1)} \right) \\ \text{subject to: } 0 \le \tau \le T, P_d = P_d^{(target)}, 0 \le P_{und} \le P_{max}, \ P_{out} = P_{out}^{(target)}, \end{aligned}$$

$$(27)$$

• For fixed power allocation and sensing parameters, solving the beamforming problem reduced to solving the problem:

maximize:
$$\mathcal{P}_0 P_{f_{\theta}} R^{(0,1)} + \mathcal{P}_1 P_d \mathbb{E}_{h_{1,2}} \left\{ R^{(1,1)} \right\}$$
, subject to: $\| \mathbf{w}_2 \| = 1$. (28)

- Given our closed-form approximation for the detection probability, for a specific value of sensing time τ we can obtain the energy detection threshold achieving the target detection probability in closed form.
- It can be proven that the rate function is then a concave function of τ and the sensing parameter optimization problem is a univariate convex optimization problem.
- As said earlier, determining P_{und} is equivalent to solving an equation which has at most one solution.
- What about the beamforming problem?

Solving the optimal beamforming problem

• Recall that:

$$\begin{split} \mathcal{E}_{h_{1,2}}\left\{R^{(1,1)}\right\} &= \log_{2}\left(\frac{\mathbf{w}_{2}^{H}\mathbf{H}_{eff}\,\mathbf{w}_{2}}{\|\mathbf{w}_{2}\|^{2}}\right) \\ &+ \frac{1}{\ln 2}\exp\left(\frac{\mathbf{w}_{2}^{H}\mathbf{H}_{eff}\,\mathbf{w}_{2}}{\mathbf{w}_{2}^{H}\frac{P_{1}\mathbf{R}_{1,2}}{N_{0}}\mathbf{w}_{2}}\right)E_{1}\left(\frac{\mathbf{w}_{2}^{H}\mathbf{H}_{eff}\,\mathbf{w}_{2}}{\mathbf{w}_{2}^{H}\frac{P_{1}\mathbf{R}_{1,2}}{N_{0}}\mathbf{w}_{2}}\right) \\ &- \frac{1}{\ln 2}\exp\left(\frac{1}{\frac{P_{1}}{N_{0}}\mathbf{w}_{2}^{H}\mathbf{R}_{1,2}\mathbf{w}_{2}}\right)E_{1}\left(\frac{1}{\frac{P_{1}}{N_{0}}\mathbf{w}_{2}^{H}\mathbf{R}_{1,2}\mathbf{w}_{2}}\right) \end{split}$$
(29)

• and:

$$R^{(0,1)} = \log_2\left(\frac{\mathbf{w}_2^H \mathbf{H}_{eff} \mathbf{w}_2}{\|\mathbf{w}_2\|^2}\right), \text{ with } \mathbf{H}_{eff} = \mathbf{I} + \frac{P_{und}}{N_0} \mathbf{h}_{2,2} \mathbf{h}_{2,2}^H.$$
(30)

• The optimal beamforming problem:

maximize:
$$\mathcal{P}_0 P_{fa} R^{(0,1)} + \mathcal{P}_1 P_d \mathbb{E}_{h_{1,2}} \left\{ R^{(1,1)} \right\}$$
 (31)

involves a composite utility function, having as arguments Rayleigh quotients (ratios of quadratic forms).

Solving the optimal beamforming problem

• Focusing on the high INR regime $(\frac{P_1w_2^HR_{1,2}w_2}{N_0}>>1):$

$$\exp\left(\frac{1}{\frac{P_1}{N_0}\mathbf{w}_2^H\mathbf{R}_{1,2}\mathbf{w}_2}\right)E_1\left(\frac{1}{\frac{P_1}{N_0}\mathbf{w}_2^H\mathbf{R}_{1,2}\mathbf{w}_2}\right) \to -C + \ln\left(\frac{P_1}{N_0}\mathbf{w}_2^H\mathbf{R}_{1,2}\mathbf{w}_2\right).$$
(32)

• $\mathbb{E}_{\mathbf{h}_{1,2}}\left\{ R^{(1,1)} \right\}$ can be closely approximated as:

$$\mathbb{E}\left\{R^{(1,1)}\right\} \approx \frac{1}{\ln 2} \ln \left(\frac{\mathbf{w}_{2}^{H}\mathbf{H}_{eff}\mathbf{w}_{2}}{\frac{P_{1}}{N_{0}}\mathbf{w}_{2}^{H}\mathbf{R}_{1,2}\mathbf{w}_{2}^{H}}\right)$$

$$+ \frac{1}{\ln 2} \exp \left(\frac{\mathbf{w}_{2}^{H}\mathbf{H}_{eff}\mathbf{w}_{2}}{\mathbf{w}_{2}^{H}\frac{P_{1}}{N_{0}}\mathbf{R}_{1,2}\mathbf{w}_{2}^{H}}\right) E_{1}\left(\frac{\mathbf{w}_{2}^{H}\mathbf{H}_{eff}\mathbf{w}_{2}}{\mathbf{w}_{2}^{H}\frac{P_{1}}{N_{0}}\mathbf{R}_{1,2}\mathbf{w}_{2}^{H}}\right) + C$$
(33)

• On the other hand:

$$R^{(0,1)} = \log_2\left(1 + \frac{P_{und} |\mathbf{w}_2^H \mathbf{h}_{2,2}|^2}{N_0}\right) = \log_2\left(\frac{\mathbf{w}_2^H \mathbf{H}_{eff} \mathbf{w}_2}{||\mathbf{w}_2||^2}\right).$$
 (34)

Solving the optimal beamforming problem

• In the presence of the constraint $\|\boldsymbol{w}_2\|^2$:

$$R^{(0,1)} = \log_2\left(\frac{\mathbf{w}_2^H \mathbf{H}_{eff} \mathbf{w}_2}{\|\mathbf{w}_2\|^2}\right)$$
(35)

• Due to concavity of the logarithm:



Figure 5: Linear approximation of $R^{(0,1)}$

with
$$\rho_{min} = \min\{eig\left(\frac{\mathbf{w}_{2}^{H}\mathbf{H}_{eff}\mathbf{w}_{2}}{\|\mathbf{w}_{2}\|^{2}}\right)\}$$
 and $\rho_{max} = \max\{eig\left(\frac{\mathbf{w}_{2}^{H}\mathbf{H}_{eff}\mathbf{w}_{2}}{\|\mathbf{w}_{2}\|^{2}}\right)\}.$
• A linear (with respect to $\frac{\mathbf{w}_{2}^{H}\mathbf{H}_{eff}\mathbf{w}_{2}}{\|\mathbf{w}_{2}\|^{2}}$) lower bound for $R^{(0,1}$ is possible.

- Using the same process, a linear lower bound for $\mathbb{R}^{(\mathscr{V},\mathscr{V}}$ can be derived.
- We can reduce the beamforming problem to a problem of the form:

$$\alpha \frac{\mathbf{w}_{2}^{H} \mathbf{H}_{eff} \mathbf{w}_{2}}{\|\mathbf{w}_{2}\|^{2}} + \beta \frac{\mathbf{w}_{2}^{H} \left(\mathbf{I} + \frac{P_{1} \mathbf{h}_{2,2} \mathbf{h}_{2,2}^{H}}{N_{0}}\right) \mathbf{w}_{2}}{\mathbf{w}_{2}^{H} \frac{P_{1}}{N_{0}} \mathbf{R}_{1,2} \mathbf{w}_{2}^{H}}$$
(36)

- We can consider finding \mathbf{w}_2 which maximizes this lower bound.
- This is a problem of maximizing the sum of two Rayleigh quotients over the unit sphere.
- Algorithms for solving such problems can be found in the literature.

The final optimization algorithm³

- 1. Find P_{und} such as to meet the target outage probability
- 2. Initialize the beamforming vectors \mathbf{w}_2 .
- 3. Solve the optimal sensing problem for the given beamformer.
- 4. Solve the optimal beamforming problem. If the improvement in the achievable rate is below a threshold, stop. Otherwise go back to step 3.

³M. C. Filippou, G. A. Ropokis, D. Gesbert and T. Ratnarajah, "Joint Sensing and Reception Design of SIMO Hybrid Cognitive Radio Systems", IEEE Transactions on Wireless Communications, vol. 15, no. 9, September 2016

Simulation scenario:

- *M* = 4 antennas at the BSs
- Outage SINR threshold $\gamma_0 = 3dB$
- Exponential correlation model with $\rho = 0.5$
- $P_{max}/N_0 = 10 dB$
- $P_1/N_0 = 10 dB$
- Unreliable sensing channel having an average SNR of -3dB
- $P_d^{(target)} = 0.975$

Comparison with conventional cognitive radio schemes



Figure 6: Rate comparison with conventional Cognitive radio schemes for $\mathcal{P}_1=0.3$



Figure 7: Rate comparison with conventional Cognitive radio schemes for $\mathcal{P}_1=0.7$

How can we apply this information availability structure in other spectrum sharing environments?

 Assuming specturm sharing between two cells, and reception using a beaformer w_i at BS_i, rate optimal combining reduces to solving the problem:

maximize:
$$\mathbb{E}_{\mathbf{h}_{j,i}}\left\{\log_2\left(1+\frac{\mathbf{w}_i^H\mathbf{h}_{i,i}\mathbf{h}_{i,i}^H\mathbf{w}_i}{N_0+\mathbf{w}_i^H\mathbf{h}_{j,i}\mathbf{h}_{j,i}^H\mathbf{w}_i}\right)\right\}$$
 (37)

- Using similar principles, the optimal beamforming at each base station may be formulated as a generalized eigenvector problem.
- Distributed user scheduling schemes can be derived, exploiting only knowledge of interference covariance information and scheduling decisions of the neighboring cell.
- Similar problems can be formulated for the downlink ⁴
- 4
- M. C. Filippou, P. de Kerret, D. Gesbert, T. Ratnarajah, A. Pastore and G. A. Ropokis, "Coordinated Shared Spectrum precoding with Distributed CSIT", IEEE Transactions on Wireless Communications, vol. 15, no. 8, August 2016
- M.C. Filippou, G. A. Ropokis, D. Gesbert, "A team decisional beamforming approach for underlay cognitive radio networks", In Proceedings of the IEEE 24th International Symposium on Personal Indoor and Mobile Radio Communications, London, Sept. 2013

Joint uplink beamforming and user schedulling in multicell networks

Optimal beamforming, exploiting the same CSI availability structure in the uplink of multicell communications systems (uniformly distributed user placements are considered). The results are plotted as a function of the SNR at the cell edge.



Figure 8: Rate comparison of mixed CSI beamforming with MRC ⁵

 $^{^5}$ M. C. Filippou, D. Gesbert, and G. A. Ropokis, "Optimal Combining of Instantaneous and Statistical CSI in the SIMO Interference channel", In Proceedings of the IEEE 77th Vehicular Technology Conference (VTC), Dresden, Jun. 2013

Applications in NOMA transmission

- We transmit to/from multiple users over the same bandwidth resources.
- Successive interference cancellation is applied in order to remove the interference caused by other users.

Example: Two user downlink NOMA



Figure 9: NOMA downlink scenario

• The BS transmits a signal of the form:

$$x(t) = \mathbf{w}_1 s_1(t) + \mathbf{w}_2 s_2(t)$$
(38)

• User; receives the signal:

$$y_{i}(t) = \mathbf{h}_{i}^{T} \mathbf{w}_{1} s_{1}(t) + \mathbf{h}_{i}^{T} \mathbf{w}_{2} s_{2}(t) + \eta_{i}(t)$$
(39)

- Since User1 is close to the BS, we can hope that the interference that it receives is strong and it can actually decoded it and cancel it out.
- User1 can then obtain an interference free version of the signal intended for it:

$$\tilde{y}_{i}(t) = \mathbf{h}_{1}^{T} \mathbf{w}_{1} \mathbf{s}_{1}(t) + \eta_{1}(t) .$$

$$\tag{40}$$

• User₂ treats interference as noise and tries to decode the message intended for it, based on the signal:

$$y_{2}(t) = \mathbf{h}_{2}^{T} \mathbf{w}_{1} s_{1}(t) + \mathbf{h}_{2}^{T} \mathbf{w}_{2} s_{2}(t) + \eta_{2}(t)$$
(41)

The (energy consumption) optimal beamforming problem

$$\begin{array}{l} \underset{\mathbf{w}_{1},\mathbf{w}_{2}}{\operatorname{minimize:}} & \|\mathbf{w}_{1}\|^{2} + \|\mathbf{w}_{2}\|^{2} \\ \text{subject to:} & \frac{|\mathbf{h}_{1}^{T}\mathbf{w}_{2}|^{2}}{|\mathbf{h}_{1}^{T}\mathbf{w}_{1}|^{2} + N_{0}} \geq \gamma_{t,2} \\ & \frac{|\mathbf{h}_{1}^{T}\mathbf{w}_{1}|^{2}}{N_{0}} \geq \gamma_{t,1} \\ & \frac{|\mathbf{h}_{2}^{T}\mathbf{w}_{2}|^{2}}{|\mathbf{h}_{1}^{T}\mathbf{w}_{1}|^{2} + N_{0}} \geq \gamma_{t,2} \end{array}$$

$$(42)$$

- The problem is a quadratically constrained quartatic programming problem in 2*M* complex random variables (M being the number of antennas).
- Using a basis for $\mathbb{C}^{M\times 1}$ which includes the vectors:

$$\mathbf{u}_{1} = \frac{\mathbf{h}_{2}}{\|\mathbf{h}_{2}\|}, \text{ and } \mathbf{u}_{2} = \frac{\left(\mathbf{I} - \mathbf{u}_{2}\mathbf{u}_{2}^{H}\right)\mathbf{h}_{1}}{\left\|\left(\mathbf{I} - \mathbf{u}_{2}\mathbf{u}_{2}^{H}\right)\mathbf{h}_{1}\right\|}, \tag{43}$$

we can recast this problem to a problem in four real variables.

• Closed form solutions can be obtained.

Jointly optimal beamforming and user pairing

- In a practical system out of the U cell users, u_i are allocated to the *i*-th chanel.
- User pairing problem: How can we split the *u_i* users in groups of two in order to apply NOMA?
- First results: Near optimal (in terms of power consumption) performance can be achieved using simple pairing rules (pairing the strongest with the weakest one.



Figure 10: Strongest-weakest user pairing

Coordinated beamforming in multicell NOMA



Figure 11: Multicell NOMA downlink scenario

- How can we optimize intercell user pairing and beamformer design decisions given statistical interference information provided by other cells?
- Instead of power minimization, how can we work towards coordinated (statistical) interference minimization?

Further research activities

- Optimal power allocation for NOMA in the presence of channel estimation errors:
 - How much power shoud we allocate for power allocation in channel estimation and how much for data transmission?
 - Rate optimal criteria
 - BER optimal criteria⁶

⁶G. A. Ropokis, M. C. Filippou, A. A. Rontogiannis, L. A. DaSilva, N. Marchetti, V. Frascolla, and P. T. Mathiopoulos, "Optimal Sensing and Power allocation in Pilot-Aided Shared Access Systems: A BER minimization apporach", 17th Intl. Workshop on Signal Processing Advances in Wireless Communications (SPAWC), Edinburgh, UK, 3-6 July 2016

Wireless powered communications

- $\bullet\,$ Optimal power allocation and beamforming for wireless powered relaying 7
- Research activities funded by the Rennes Metropole foundation.

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- G. A. Ropokis, N. Marchetti, and L. A. DaSilva, "Cooperative Beamforming Exploiting Energy Recycling", presented at ICT 2018, Saint Malot, France, June 2018 [.pdf].
- G. A. Ropokis, M. M. Butt, N. Marchetti, and L. A. DaSilva, "Optimal Power Allocation for Energy Recycling Assisted Cooperative Communications" IEEE WCNC, San Francisco, USA, March 2017

G. A. Ropokis, "Multi-relay cooperation with self-energy recycling and power consumption considerations", presented at the 6th International Workshop on Cooperative Wireless Networks, October 2019

Teaching and supervision activities

Participation in lectures for courses on

- Digital Communications
- Wireless Communications
- Wireless Networks
- Optimization
- Preparation of laboratory exercises on wireless networking using python.

CentraleSupélec

Participation in lectures/laboratory sessions for courses on:

- Digital and Wireless Communications
- Cognitive Radio
- Machine Learning applications in wireless communications
- Probability Theory
- Linear Algebra

Key responsibilities:

- Professor in charge of the course on Statistical Signal Processing (2018-2019).
- Professor in charge for the course of Optimization
- Professor in charge for pedagogical activities for introducing students to research professions.

PhD student supervision:

- Youssef Fakih
- Georgios Konstantopoulos

Having a Computer Engineering and Informatics background, I would be happy to participate in courses related to:

- Numerical Analysis
- Data bases
- Data structures
- Algorithms and Complexity

Thank you for your attention