A Modeling Framework for Analyzing European Balancing Markets

Anthony Papavasiliou, Gilles Bertrand
CORE, UCLouvain

Technical University of Crete, 2021

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Outline

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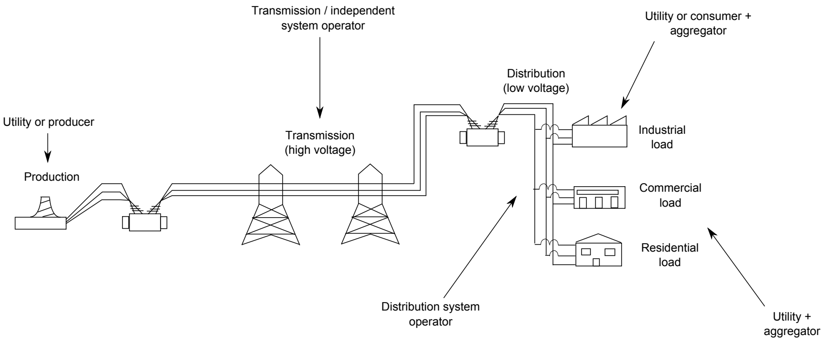
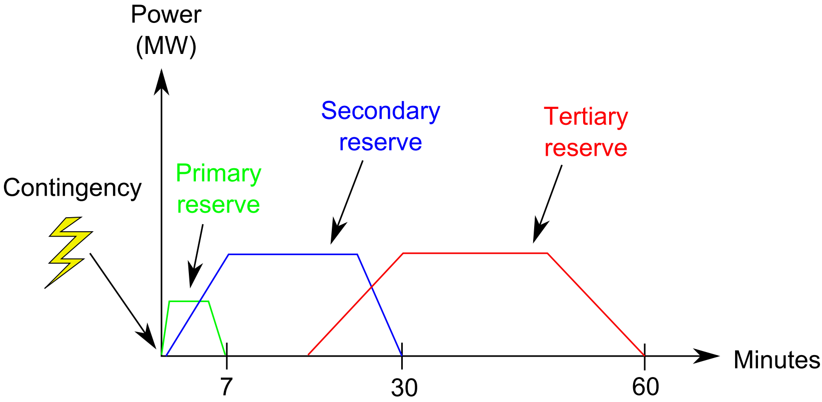
Actors

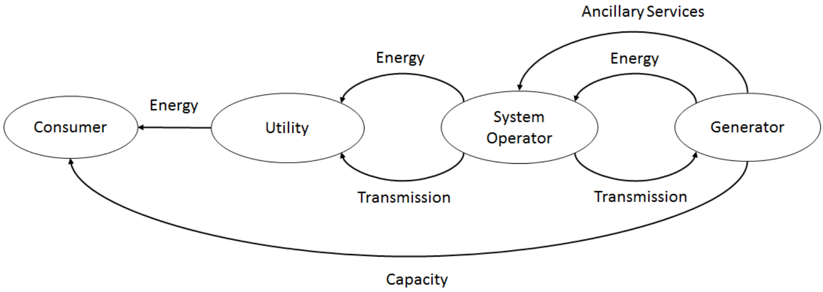


Transmission / independent system operator
Utility or consumer + aggregator
Distribution (low voltage)
Utility or producer
Transmission (high voltage)
Industrial load
Production
Commercial load
Residential load
Distribution system operator
Utility + aggregator

Sequential Activation of Reserves



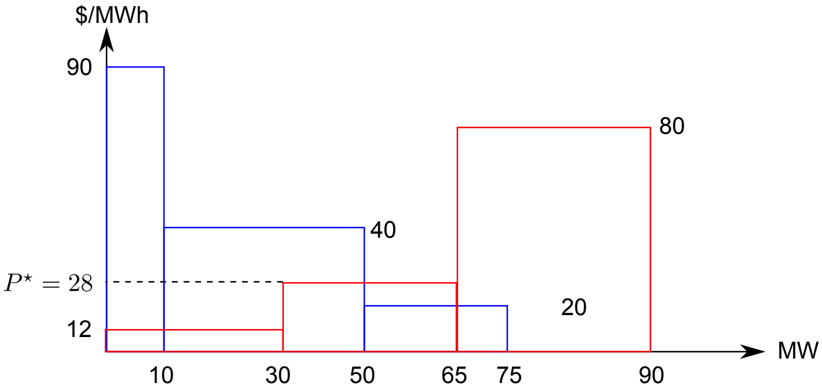
Blueprint of an Electricity Market



Uniform-Price Auctions

- Generator bids: price-quantity pairs (P, Q), representing price P at which suppliers are willing to produce quantity Q
- Consumer bids: price-quantity pairs (P, Q) representing price P consumers are willing to pay for quantity Q
- Obligations and payoffs
  - Market clearing price $P^\star$: intersection of supply and demand curves
  - *In the money* supply bids: produce and receive $P^\star$ \$/MWh
  - *In the money* demand bids: consume and pay $P^\star$ \$/MWh

Uniform Price Auctions



- Uniform price auctions aim to approximate second-price auctions (with their associated appealing incentive compatibility properties)
- Uniform price auctions are the *standard* mechanism for trading energy and services in electricity markets

Greek Nomenclature

Balancing market	Αγορά εξισορρόπησης
Contingency	Απρόβλεπτο συμβάν
Frequency restoration reserves (FRR)	Εφεδρείες αποκατάστασης συχνότητας (ΕΑΣ)
Balancing service provider (BSP)	Πάροχος εφεδρείας
Balancing responsible party (BRP)	Φορέας με ευθύνη εξισορρόπησης
Nominated electricity market operator (NEMO)	Διορισμένος διαχειριστής ηλεκτρικής ενέργειας
Transmission system operator (TSO)	Διαχειριστής εθνικού συστήματος μεταφοράς ηλεκτρικής ενέργειας
Shortage / scarcity pricing	Μηχανισμός τιμολόγησης σε ανεπάρκεια

US/EU Nomenclature

Balancing capacity <-> day-ahead / forward reserve capacity

Balancing energy <-> real-time energy

aFRR, mFRR <-> operating reserves (resources with a response time of seconds to minutes)

Outline

Trading of Energy and Reserve in EU Markets

- Important EU-wide balancing market integration initiatives
- Functional separation:
  - TSOs: forward procurement of reserve capacity, deployment of reserve capacity in real time
  - NEMOs: operation of day-ahead and intraday market
- Balancing Responsible Parties (BRPs): *price-inelastic* buyers or sellers of real-time energy
- Balancing Service Providers (BSPs): *price-elastic* suppliers or consumers of real-time energy
  - BSPs commit to bidding at least DA reserve capacity to RT balancing markets
  - Each BSP must be attributed to at least one BRP portfolio, according to EU law (EBGL)
- BRPs and BSPs face a different price for real-time energy:
  - BRPs: imbalance price
  - BSPs: balancing price

Role of ORDC in a Regime of Renewables

- Accurate valuation of energy and reserve capacity is an increasingly crucial function of RT markets in a regime of large-scale renewable energy integration
- Operating reserve demand curves (ORDCs) [Hogan, 2005]: means for achieving this goal
  - ORDC adders computed on basis of available reserve capacity in the system
  - When reserve capacity decreases, ORDC adders increase (value of reserve in tight system)
  - When reserve capacity increases, ORDC adders dissipate

Scarcity Pricing Evolutions Internationally

- ORDC adders have been adopted in Texas
- Adoption of ORDCs is moving forward in PJM
- European Commission Electricity Balancing Guideline article 44(3)

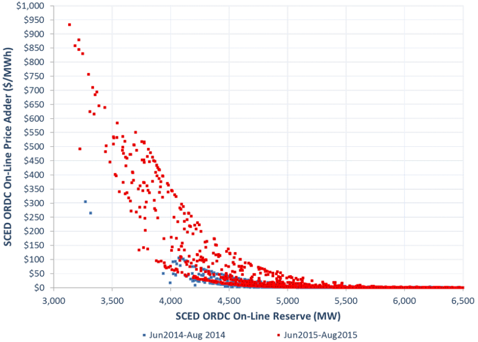


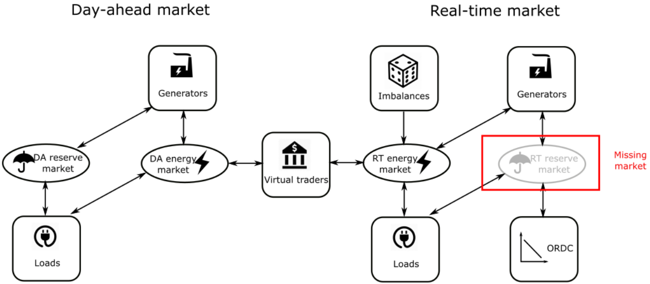
Figure: ORDC adders in Texas, 2014-2015

Scarcity Pricing Evolutions in Belgium

- Preliminary analyses [Papavasiliou, 2017], [Papavasiliou, 2018] focused on quantifying possible implications of mechanism for reserve resources
- Belgian system operator [ELIA, 2018] publishes scarcity adders based on the “available reserve capacity” (ARC) of the system
- Since October 2019, ELIA publishes scarcity prices for information purposes
    - Computed for every quarter of the day
    - Published one day after operations
- ELIA public consultation on scarcity pricing [ELIA, 2020]

Translating First Principles to the EU Design

- ORDC essentially sets a RT price for reserve
- In equilibrium, energy and reserve prices follow each other in lock step



So what does it mean to introduce ORDC adders to the EU market, if we do not have a RT market for reserve?

- Adders to the imbalance price (BRPs)?
- Adders to the balancing price (also BSPs)?
- What about RT reserve *capacity*?

Our Proposal for Implementing Scarcity Pricing

- **Proposal 1**: introduction of a scarcity adder to the imbalance price
- **Proposal 2**: application of same adder to the balancing energy price
- **Proposal 3**: implement a real-time market for reserve capacity (equivalently, market for reserve imbalances, in the same way that we operate a market for energy imbalances)

Stochastic Equilibrium Models

- Rationale of our proposal:
  - **Law of one price** [Cramton, 2006] applied to real-time energy
  - **Back-propagation** of reserve value: If we put in place a real-time market for reserve capacity, agents will only sell reserve capacity in forward markets at the value that they would need to buy it back in real time
- Stochastic equilibrium [Papavasiliou, 2020]
  - Can be used to understand effect of certain market design choices on back-propagation ...
  - ... but it embeds the law of one price as an *assumption*

Markov Decision Processes

- Our approach in this work: represent balancing market as a Markov Decision Process (MDP)
- Growing body of work in this direction
  - Early work: analysis of design changes on English and Welsh markets [Bower, 2001], [Bunn, 2001]
  - Application of Q-learning [Naduri, 2007], [Yu, 2010]
  - Deep learning [Ye, 2019], [Ye, 2020]

A Caveat About MDP Models

- MDP framework: powerful modeling flexibility ...
- ... but difficult to extract generalizable conclusions
- We supplement our MDP-based market simulation framework with analytical results under **perfect competition**

Outline

Generic Agent in Our Model

We consider a *general* agent participating in the balancing market as one which owns

1. Uncontrollable assets
2. Controllable assets (reserves)
   - marginal cost C
   - upward capacity P^+
   - downward capacity P^-

Single-Stage MDPs

- Agent that decides how much balancing energy q to offer to a uniform price auction with constant price λ^B
  - Action of the agent: quantity q:
  - Reward: $(\lambda^B - C) \cdot q$, with qa the matched quantity
- Agent submitting price-quantity pairs
  - Action space: (p, q), i.e. offer of q MW at p €/MWh
  - If bids of competitors are fixed, this implies a balancing price
  - Reward of the agent: $(\lambda^B - C) \cdot qa$
- System-level uncertain imbalance $\Rightarrow$ uncertainty in balancing price

Differentiating BRP and BSP Settlement

Belgium applies a surcharge α^U whenever the system is short, or a discount α^L whenever the system is long:

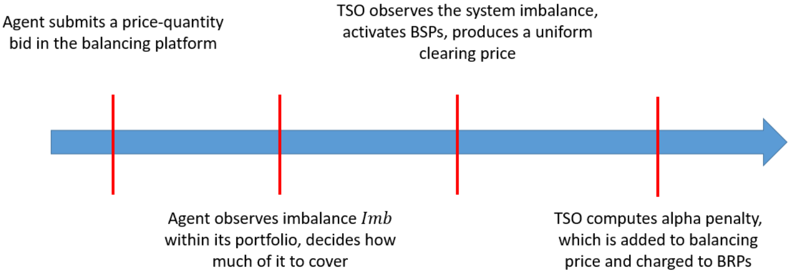
$$\begin{aligned} \lambda^I &= \lambda^B + \alpha \\ \alpha &\triangleq \alpha^U \cdot \mathbb{I}[Imb^t > UI] - \alpha^L \cdot \mathbb{I}[Imb^t < LI] \end{aligned}$$

Notation:

- λ^I: imbalance price
- Imb^t: total imbalance of the system
- UI and LI: upper and lower imbalance thresholds at which the surcharge or discount apply, respectively

Actual formula used in Belgium is more complex in practice (accounted for in simulations)

Two-Stage MDPs: Stage 1

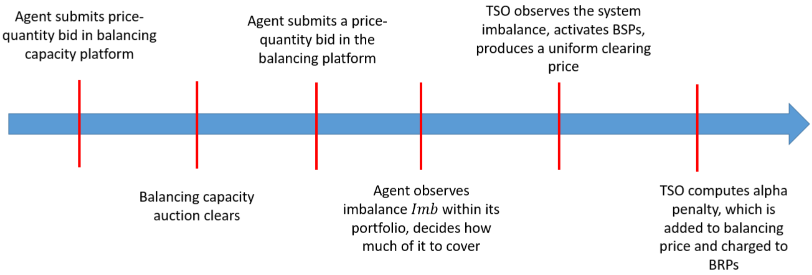


- Action: (p, q), price-quantity offer in balancing platform
- No reward is collected at this stage.

Two-Stage MDPs: Stage 2

- State:
  1. bid price p
  2. leftover BSP capacity after some capacity has been offered to the balancing auction
  3. imbalance Imb of an agent
- Action: How much of the imbalance Imb to cover ("active imbalance", must be limited to leftover capacity that BSP has not allocated to reserve auction)
- Reward:
  1. BSP payment for upward/downward activation, $\lambda^B \cdot qa$
  2. BRP payment for imbalance settlement, $-\lambda^I \cdot (Imb - ai)$
  3. fuel costs related to self-balancing and BSP activation, $-C \cdot (ai + qa)$

Three-Stage MDPs: Stage 1



- Stage 1
  - Action: (p^R, q^R), price-quantity offer in balancing capacity auction
  - Rewards: payment from balancing capacity auction

Three-Stage MDPs: Stages 2, 3

- Stage 2
  - State: capacity qa^R awarded in balancing capacity auction
  - Action: (p, q), the price-quantity offers in balancing platform, with $q \geq qa^R$
- Stage 3: identical to stage 2 of two-stage MDP

Considered Designs

- **Option D1**: vanilla balancing market design

$$\lambda^B \cdot qa - \lambda^B \cdot (Imb - ai) - C \cdot (qa + ai)$$

- **Option D2**: imbalance price adders (current Belgian market)

$$\lambda^I = \lambda^B + \alpha$$

- **Option D3**: Scarcity adders limited to imbalance prices [ELIA, 2020]

$$\lambda^I = \lambda^B + \lambda^R$$

- **Option D4**: Real-time market for balancing capacity

$$(\lambda^B + \lambda^R) \cdot qa - (\lambda^B + \lambda^R) \cdot (Imb - ai) - C \cdot (qa + ai)$$
$$+\lambda^R \cdot (P^+ - qa - ai) - \lambda^R \cdot qa^R$$

Outline

Perfect Competition Assumption

Perfect competition assumption: We consider fringe agents, i.e. ones with infinitesimal capacity who do not influence price outcomes

Rationale of assumption:

1. Unveiling difficulties in back-propagating reserve prices in the case of perfect competition suggests *fundamental market design problems*
2. Analytical results from perfect competition assumption allow better understanding / interpretation of MDP results

Statement of Analytical Results: D1

1. It is optimal for agents to bid their entire balancing capacity at the true marginal cost to the balancing auction
2. For agents with upward balancing capacity ($P^+ > 0$), the opportunity cost of bidding their capacity to the day-ahead reserve auction is zero
3. This is a pure strategy Nash equilibrium

Statement of Analytical Results: D2

1. Under the assumption of independent symmetric imbalances, it is optimal for agents to bid their entire balancing capacity at the true marginal cost to the balancing auction
2. For agents with upward balancing capacity ($P^{+} > 0$), the opportunity cost of bidding their capacity to the day-ahead reserve auction is zero
3. This is a pure strategy Nash equilibrium

Statement of Analytical Results: D3

1. For sufficiently high-cost agents, is it optimal for agents for them to bid their entire balancing capacity at the true marginal cost to the balancing auction
2. For agents with upward balancing capacity ($P^+ > 0$), the opportunity cost of bidding their capacity to the day-ahead reserve auction is less than or equal to the scarcity value $\mathbb{E}[\lambda^R]$
3. This does *not* characterize a pure strategy Nash equilibrium, since some agents find it optimal to self-balance

D3 depresses scarcity price in two ways:

1. Agents who find it optimal to bid their entire capacity to the balancing auction face an opportunity cost of zero for bidding reserve in the day ahead
2. Agents who self-balance depress balancing energy prices

Statement of Analytical Results: D4

1. It is optimal for agents to bid their entire balancing capacity at the true marginal cost to the balancing auction
2. For agents with upward balancing capacity ($P^+ > 0$), the opportunity cost of bidding their capacity to the day-ahead reserve auction is the scarcity value $\mathbb{E}[\lambda^R]$
3. This is a pure strategy Nash equilibrium

Among the analyzed options, ($D4$) is the only option which

- back-propagates the real-time value of reserve capacity to day-ahead reserve auctions, while
- preserving the incentive of agents to make their balancing capacity available in the balancing market

Proof for (D1)

- Without loss of generality, consider agent which only has downward capacity (i.e. $P^+ = 0$ and $P^- < 0$) or only upward capacity (i.e. $P^- = 0$ and $P^+ > 0$)
- Fringe assumption implication: no influence of imbalance on expected imbalance price $\Rightarrow D \triangleq -\mathbb{E}[\lambda^B \cdot Imb]$ is a constant offset to the imbalance payoff of the agent
- Two possible suppliers:
  - Cheap: $\mathbb{E}[\lambda^B] \geq C$
  - Expensive: $\mathbb{E}[\lambda^B] < C$
- In what follows, we focus on cheap suppliers with upward capacity ($\mathbb{E}[\lambda^B] - C \geq 0, P^+ > 0, P^- = 0$)

Proof for (D1): Imbalance Payoff z_I

Imbalance payoff:

$$\max_{ai} (\mathbb{E}[\lambda^B] - C) \cdot ai - \mathbb{E}[\lambda^B \cdot Imb]$$
$$ai + q \leq P^+$$
$$ai \geq 0$$

We have $ai^\star = P^+ - q$, expected payoff z_I is:

$$z_I = (\mathbb{E}[\lambda^B] - C) \cdot (P^+ - q) + D$$

Proof for (D1): Balancing Market Payoff z_B

Balancing payoff $z_B(\omega)$:

- Out of the money: if $p > \lambda^B$, then $z_B(\omega) = 0$
- At the money: if $p = \lambda^B$, then $z_B(\omega) = (\lambda^B - C) \cdot qa$ for some qa which selected by the auctioneer; use fringe assumption to set $qa = 0$ and $z_B = 0$
- In the money: if $p < \lambda^B$, then $z_B(\omega) = (\lambda^B - C) \cdot q$

- Balancing payoff $z_B(\omega)$ is random, depends on system imbalance
- Denote probability measure of balancing price λ^B as μ
- Expected balancing market payoff:

$$\begin{aligned} z_B &= \mathbb{E}[z_B(\omega)] \\ &= \int_{x>p} (x - C) \cdot q \cdot d\mu(x) \end{aligned}$$

Proof for (D1): Optimal Balancing Market Price p

Overall agent payoff:

$$\begin{aligned} R(p,q) &= z_I + z_B \\ &= C_1 - C_2 \cdot q + C_3(p) \cdot q \end{aligned}$$

where:

$$\begin{aligned} C_1 &= (\mathbb{E}[\lambda^B] - C) \cdot P^+ + D \\ C_2 &= \mathbb{E}[\lambda^B] - C \\ C_3(p) &= \int_{x>p} (x - C) \cdot d\mu(x) \end{aligned}$$

For given balancing quantity bid q, first-order conditions with respect to p are:

$$\begin{aligned} \frac{\partial R(p,q)}{\partial p} &= C_3'(p) \cdot q \\ &= -\mu(p) \cdot (p - C) \cdot q \end{aligned}$$

Proof for (D1): Optimal Balancing Market Price p

Payoff function $R(p, q)$ for fixed q is

- increasing in $(-\infty, C]$
- zero at C
- decreasing in $[C, +\infty)$

Thus, for any q, an optimal strategy is to bid the true cost, which implies

$$R(C, q) = C_1 - C_2 \cdot q + C_3(C) \cdot q$$

Proof for (D1): Optimal Balancing Market Quantity q

First-order conditions with respect to q:

$$
\begin{aligned}
\frac{\partial R(C,q)}{\partial q} &= -C_2 + C_3(C) \\
&= -(\mathbb{E}[\lambda^B] - C) + C_3(C) \\
&= -(\int_{x \leq C} (x - C) \cdot d\mu(x) + \int_{x > C} (x - C) \cdot d\mu(x)) \\
&\quad + \int_{x > C} (x - C) \cdot d\mu(x) \\
&> 0
\end{aligned}
$$

Therefore, it is optimal to bid $q^\star = P^+$ in the balancing auction, and $ai^\star = 0$

Proof for (D1): Optimal Balancing Market Quantity q

- When being in active imbalance, agent takes risk of producing power when being out of the money
- Instead, balancing market will only activate agent when its marginal cost is lower than the balancing price
- When the balancing and imbalance price are equal, the agent has the incentive to bid its entire capacity to the balancing auction

Proof for (DA): No Back-Propagation

- Every MW cleared in a forward reserve auction comes with an obligation to bid that MW in the balancing auction
- This is profit lost in the balancing and imbalance phase
- Since the optimal strategy of the agent is to anyways bid its entire capacity in the balancing auction, there is no opportunity cost for the agent, i.e. $dR^\star/dq = 0$

Outline

Validation Settings

- Fringe supplier
  - Fringe supplier: $P^+ = 1$ MW, $P^- = 0$ MW
  - Marginal cost: $C = 50$ €/MWh
  - Balancing auction bid q and reserve auction bid q^R is either 0 MW or 1 MW
  - Agent can bid any value p between 25 to 75 €/MWh, in increments of 5 €/MWh
- Imbalances:
  - System imbalance $\sim N(0, 91.5)$
  - Fringe agent imbalance: $\sim N(0, 0.41)$
- Balancing supply function:
  - $a + b \cdot q$, with $a = 50$ €/MWh, and $b = 0.11$ (€/MWh)/MW
  - Approximation (for analytical solution purposes) of a balancing market with 8 agents (see next slide)

Validation Settings

We validate our analytical results using the MDP model

- We assume a fringe agent
- We validate all designs
- See appendix for detailed assumptions of validation study

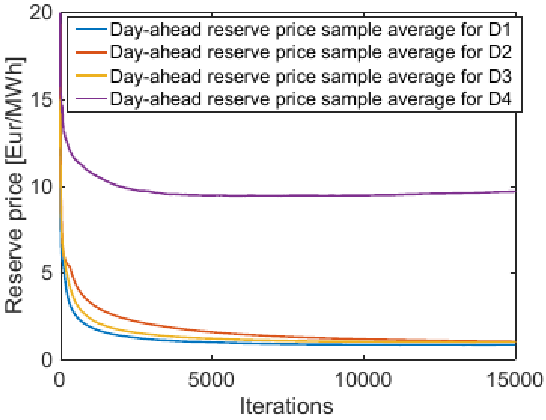
Multi-Agent Simulation Settings

- Discretize agent action space by having agents bid in price increments of 5 €/MWh and in quantity increments of half of their capacity
- Each agent is facing a portfolio imbalance which is uniformly distributed between zero and half of its capacity
- System imbalance: zero mean and standard deviation of 21.9 MW
- Agent imbalances are independent of each other and system imbalance
- Day-ahead reserve demand curve identical to real-time reserve demand curve

Multi-Agent Learning Settings

- Q-learning algorithm using $\epsilon-$greedy policy, with ϵ_k evolving as $\frac{0.05}{N-k}$
- All agents are learning simultaneously $\Rightarrow$ no convergence guarantees
- We run $1,500,000$ iterations in blocks of 100
- After each block of 100 iterations, we compute the outcome that we would have obtained in the reserve market if each agent were applying its policy greedily

Multi-Agent Results



Multi-Agent Observations

- For ($D3$), the reserve price sample average arrives slightly above the one resulting from ($D1$): certain low-cost producers may face a positive opportunity cost when bidding into the day-ahead reserve market
- Under design ($D4$), the day-ahead reserve price converges to a value which is close to the average real-time scarcity adder, i.e. 9.4 €/MWh

Conclusions and Perspectives

Conclusions:

- MDP is an interesting framework for analyzing market design options, when supplemented by analytical results
- A market for balancing capacity imbalances can
  1. back-propagate the value of reserve capacity to forward reserve markets
  2. while also preserving incentive of agents to offer their capacity in the balancing market

Perspectives:

- Collaboration with CREG on calibration of ORDC to Belgian system needs
- Discussions with ELIA on scarcity pricing proposal [ELIA, 2020]
- Address questions of market stakeholders on public consultation of ELIA
- Further clarify interaction of market design proposal with EU legislation

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Thank You for Your Attention

For more information:

- anthony.papavasiliou@uclouvain.be
- https://ap-rg.eu/
- https://perso.uclouvain.be/anthony.papavasiliou/public_html/

Vanilla EU Design (D1)

- Default design: imbalance penalty α of Eq. (1) is equal to zero
- Balancing price equals the imbalance price, $\lambda^I = \lambda^B$
- Compatible with EBGL
- Failure to generate a forward reserve price signal

Imbalance Penalties (D2)

- Belgian government claims that the imbalance penalty α "already exhibits quite some characteristics of a scarcity pricing mechanism"
- In case of independent imbalances and a symmetric imbalance penalty α, design $(D2)$ is shown to behave identically to design $(D1)$
- Design $(D2)$ relies on imbalance penalties α which depend on level of system imbalance, not to be confused with level of scarcity in the system
- In practice, imbalance alpha depends on imbalance of the current and previous interval $\Rightarrow$ MDP model requires an additional state variable, imbalance of previous balancing interval (added to state vector of stages 2 and 3)

Adders on Imbalance Charges (D3)

- ORDC adder:

$$\lambda^R = (VOLL - \lambda^B) \cdot LOLP(P^{+,tot} - Imb^t) \cdot \mathbb{I}[P^{+,tot} - Imb^t \geq 0] + (VOLL - C^{max}) \cdot \mathbb{I}[P^{+,tot} - Imb^t < 0]) \quad (1)$$

  - *VOLL*: estimate of value of lost load
  - $P^{+,tot}$: total reserve capacity
  - $LOLP(\cdot)$: loss of load probability as a function of available reserve capacity
  - C^{max}: estimate of marginal cost of marginal unit
- ELIA proposal: apply λ^R as an imbalance charge
- This produces a forward reserve price that is significantly weaker than the average value of balancing capacity to the system

Scarcity Pricing (D4)

- Replace α with λ^R in Eq. (1)
- Introduce the following term in settlement:

$$-\lambda^R \cdot qa^R + \lambda^R \cdot (P^+ - qa - ai)$$

- Second term induces agents to bid reserve capacity in forward markets in a way that anticipates expected price at which they would be required to buy that reserve capacity back in real time $\Rightarrow$ **back-propagation**
- D4 implements an imbalance mechanism for balancing capacity / RT market for reserve capacity (analogous to imbalance mechanism for balancing energy / RT energy market)
- Compatible with article 20 of Clean Energy Package
- We need to add awarded day-ahead reserve capacity qa^R to state of third time step of MDP model (since it affects third-stage payoff)

MDP Model Supply Function

Fringe agent that we are interested in is agent A5

	A1	A2	A3	A4	A5	A6	A7	A8
P^+	0	0	0	0	1	100	100	100
P^-	-100	-100	-100	-50	0	0	0	0
C	20	30	40	50	50	60	70	80

Table: Units are in [MW] for P^+ and P^-, and in [€/MWh] for C.

Other Settings

- For design $(D2)$, we use ELIA formula: $UI = LI = 150$ MW, and
$$\alpha^U = \alpha^L = \frac{200}{1 + \exp\left(\frac{450 - x}{65}\right)}$$
where $x = \frac{|Imb^t| + |Imb^t_{t-1}|}{2}$ is the average of the absolute total system imbalances of the previous and current imbalance interval
- For design $(D3)$ and $(D4)$, we assume $VOLL = 920$ €/MWh
- Q-learning algorithm
  - Learning rate: $\frac{1}{n(s,a)}$ for each state-action pair (s, a), where $n(s, a)$ counts the number of visits to (s, a)
  - We run $2,000,000$ episodes for each design with the same seeds, in order to isolate the effect of the market design changes on the results

Analytical Solution

Design	$(D1)$	$(D2)$	$(D3)$	$(D4)$
q^* [MW]	1	1	0	1
p^* [€/MWh]	50	50	any	50
Average Profit [€]	4.04	4.04	12.57	16.63
Opportunity cost $dR^\star/dq$ [€]	0	0	8.53	12.59

Table: Analytical Solution

MDP Solutions

Design	$(D1)$	$(D3)$	$(D4)$
q^* [MW]	1	0	1
p^* [€/MWh]	55	any	50
Average Profit [€]	6.34	14.43	18.85
Opportunity cost $dR^\star/dq$ [€/MWh]	0	8.11	12.71

Table: MDP results for $(D1)$, $(D3)$ and $(D4)$

Imb_{t-1}^t [MWh]	$(\infty, -150]$	(-150,0]	(0,150]	$(150, \infty)$
q^* [MW]	1	1	1	1
p^* [€/MWh]	50	55	55	50
Average Profit [€]	6.43	6.30	6.32	6.46
$dR^\star/dq$ [€/MWh]	0	0	0	0

Table: MDP results for $(D2)$

Validation Observations

- For every design, the bid quantity and price are equivalent for the analytical case and the MDP model
- Profits are in the same range for the analytical solution and the MDP model
- Opportunity costs are very close to each other for the analytical model and the MDP code
- For design ($D2$), the range of values in the imbalance of the previous period, Imb_{t-1}^{t}, does not influence the selected action or the profit, in line with analytical results